feedback webinar on ‘the framework’ (2023)

(automated transcription)

This feedback webinar will be about the second module of the course, which is about the general framework for hyperfine interactions. But before we come to that point, let's go one step back to last week. I told you last week there were not too many questions in the forum, actually almost nothing. But one question came later, and so let's go back to that part and go to that question. It was at the stage where we were calculating the classical energy, the classical rotation energy and rotation frequency for a nucleus. And somebody was wondering, yeah, I understand this formula, but does it make sense to approximate a nucleus by a classical sphere, because we know the nucleus is actually a wave function and you need all this machinery of quantum mechanics to discuss a nucleus. So how does the sphere make sense? I found this an interesting comment, because it's not what you usually hear. Usually you hear the opposite. People are asking for pictures to understand these weird quantum physical systems better in a classical way. So here it's the other way around. What could I answer to that? There is the historical development that we cannot neglect. So these phenomena like angular momentum, spin for quantum particles, they have an origin in classical physics where people were dealing with particles that can, for instance, spin around one of their symmetry axes. And it's because the quantum particles had some observable properties that had at least some features in common with this spinning that the observable property that we now call spin is called spin. And therefore, although it is not really a nucleus that is spinning, there are energy levels that can be converted in frequencies that can be attached to a classical particle that is spinning with that frequency. It's not what really happens. You cannot visualize it that way, but historically it comes from that picture. And therefore, it is a legitimate question, what would be the rotation frequency of a classical particle that has an energy, rotation energy, that is the same as the energy you can associate to a nucleus that is spinning or that has this value of spin. So it's mixing two pictures. I'm totally aware of that. But when discussing these topics, these two pictures are both used, the classical picture or the semi-classical picture and the quantum picture. And sometimes it's insightful to jump back and forth between these two. Good, that was about last week. Until this week, there were almost no questions that arrived. So once again, just like last week, I try to encourage you, please, if you run into a question, submit it, it will be dealt with in the feedback webinar. This is your chance to ask questions, the main opportunity. I hope this is just a good sign that you digest everything without problems and then I'm totally happy. And if you run into a question, please, please, please post it. So the first topic of the past week was this very important picture number one, which summarizes the scenery we will play with, all the energy levels of the system, the general atom that we will be talking about. All our hyperfine interactions will take place in this landscape. And I asked you about this very important picture number one in order to have a feeling for the relative energy scales. If the arrow, well, that in the slide, yeah, if the arrow, the micro EV arrow at the very right-hand side, if that one would be five millimeters long, how long would be the arrow at the very left-hand side, this kilo electron volt to mega electron volt arrow. And that was a straightforward exercise. You made some calculations and the conclusion was if we start from five millimeters at the left-hand side, then at the right-hand side, then at the left-hand side, we would end up with 5,000 kilometers about the radius of the earth. If we talk about kilo electron volts or 5 million kilometers about the distance, 10 times the distance from the earth to the moon, if we talk about mega electron volts. Just to emphasize that there is really, really, really a large order of magnitude difference between the hyperfine levels and the nuclear levels. And everything in between has, you see, millielectron volt, the fine structure, that's five meters on this scale. Electron volts, the electronic energy levels, that's five kilometers on this scale. So the order of magnitude difference is there. The confidence question that relates to this very important picture number one is I can list the different energy scales in a system of electrons and nuclei and I can relate them to the relevant physical processes. So you seem to be quite confident about that and if you are able to explain for the picture that you see now on the screen what are the underlying physical processes, then you have understood this picture completely. This picture is so important that I invited you to print it and to put it in a place where you can see it every day in order for the next few weeks to really ingrain that picture in your brain. Once again, I noticed that you seem to be a group of people who are a bit shy because I didn't receive any pictures of where you have put that picture in your environment, therefore I cannot show any of these to you. But I can show you some pictures of the previous years, a wall where somebody taped this picture or there was somebody who made it as the wallpaper on the laptop or as a little icon on the laptop such that you can immediately open and access that picture or in the kitchen above the hood and one I really liked above the baby changing table in a nice frame even. So I know young parents they are at that baby changing table many many times a day, so if you hang the picture there then for sure you will see it many times a day and I bet this student will never forget this picture for the rest of his or her life. So if this inspires you, if you find a nice place for that picture, put it there, you will not regret it, it will really help to make this picture something that is very very familiar to you and it's worth understanding that picture, this will be your guide, your compass through the hyperfine physics world. Good, before coming to hyperfine structure we had to make a quick stop at the fine structure level which is basically discussing the physics of spin-orbit coupling, this is a topic that many of you certainly the ones with a physics background have seen in other courses so we will not spend too much time to that and I asked you and I see I forgot to put the real question here what was the question yeah that spin, the spin you see in spin-orbit coupling that spin is often said to be a relativistic effect, spin-orbit coupling itself is said to be a relativistic effect. So where does that come from how can you understand that spin-orbit coupling is a relativistic effect and you had to answer this in a forum I show here some of your answers first two answers that are not valid and some people thought well this is because we have to deal with the speed of light electrons are moving at large speeds considerable fraction of the speed of light so therefore this is relativistic. Okay it is true that the electrons move at a high speed and indeed you have to take this high speed into account in order to adjust the mass of the electron for instance an electron will have a larger mass than its rest mass due to its high speed but that itself is not directly leading to spin-orbit coupling there are two ways in which we can understand how spin-orbit coupling is relativistic and here I show two other answers that touch the first argument the spin from spin-orbit coupling that is something that spontaneously evolves from the Dirac equation which is the relativistic version of the Schrodinger equation. In the Dirac equation spin is a natural concept and therefore spin has to be seen as a relativistic effect and therefore also spin-orbit coupling I can summarize this in this little diagram the Schrodinger equation that does not know about spin that does not know about relativistic effects that does not know about spin-orbit coupling but you can manually improve the Schrodinger equation and you can add spin to that as an externally given variable so then it knows about spin the full relativistic effects are still not there there is an intermediate level the scalar relativistic level where some of these effects can be integrated in the Schrodinger equation but certainly not fully relativistic and if you want to add spin-orbit coupling as the major missing relativistic effect you can do that with a perturbation treatment of that changed Schrodinger equation. But it is only when you go to the Dirac equation there everything is fully integrated spin relativity and therefore also spin-orbit coupling are naturally emerging from the Dirac equation. So that is one way how we can see that spin-orbit coupling is a relativistic effect. The other way and that is touched in this answer here I will not read the entire answer but I have highlighted that this is about the Lorentz transformations so let's go through that you might remember the classical example from Lorentz transformations if you have a static charge and a magnet in a train then the magnet and the charge do not interact but if the train is moving and you observe that system from outside the train then you see the moving charge you see a current and a current and a magnet they do interact. So for you outside the train they are interacting for an observer in the train they are not interacting how can we reconcile that and that can be done with the Lorentz transformation where you can understand that these electromagnetic fields transform not in the classical way and they will transform in exactly the way that is needed to resolve your contradiction. Exactly the same happens in an atom and I draw here a cartoon for a simple one electron atom where the electron is orbiting so it has orbital angular momentum in the classical sense the way how it is drawn here is a planetary orbit so orbital angular momentum can visualize it that way that the electron also has spin which is the quantum phenomenon spin I like to represent by a bar magnet because then you can see that there is a magnetic field around the electron due to its spin and what we now wonder about is how is that bar magnet how is the spin oriented with respect to the angular momentum vector of the orbit and if you just take a spin one so which can have three different orientations and this we can wonder is it up is it parallel or is it down. How can you calculate that the reasoning goes like this we put ourselves at the position of the electron so from the position of the electron it looks like the nucleus is orbiting the electron we have our bar magnet and the nucleus the orbiting nucleus generates a current and therefore a magnetic field and so my magnetic field interacts with the bar magnet of the electron so these two motions are coupled spin and orbit are coupled there we have the world and there will be a lowest energy orientation but in order to do this transformation this change of axis system from an axis system connected to the lab or connected to the nucleus to an axis system connected to the electron there you need the Lawrence transformations to do this in a proper way and that is why you have to deal with spin orbit coupling in a relativistic frame. So that is a second reason why we call spin orbit coupling a relativistic phenomenon and the last question I asked you about this topic was that the fine structure so this is still about the fine structure that this is something that is technologically important the old style highway lights this orange light and this is from sodium lamps and these use an atomic transition where an fine structure is involved and I asked you to search some information about it and to report and I put here two of your answers it's mainly for the links so if you didn't find these links if you found other links you can inspect what can be read here at these various websites and I summarize the story so it's about sodium so one electron in outside the noble gas core that can either be in an s orbital or in a p orbital and it can choose and you can choose then how is the spin of that single electron oriented with respect to the orbital angular momentum in that s or p orbital there is no orbital angular momentum in the s orbital so there you have no choice but in the p orbital there are two possibilities and therefore the upper two levels in this diagram they do not have exactly the same energy and that energy splitting which is a fine structure splitting that is exactly due to that spin orbit coupling the spin of the single electron is oriented in two different ways with respect to the orbital angular momentum and therefore the light that is emitted when an electron decays or when the system decays from that p state to the s state there will be two energies involved very close to each other only thousands of an eV different from each other but if you have enough resolution as you see on that oscilloscope picture at the left hand side then you see that these are indeed two different lines and this is here the cartoon version of this so the question I asked is we can wonder in that lower level the one that is blinking there which of these three situations is true where how is the electron pointing we will come to an answer with this a specific answer but before we can answer that we need to know how term symbols in atoms are used and I asked you how familiar you are about term symbols so there seems to be a hesitation there many people feel very familiar about it but not everybody so let us recapitulate how for sodium we can come to these term symbols where you see here the general description and we first have to fill we have to find s the total spin and then in the term symbol this is written as 2s plus one there is only one electron so spin one half twice one half plus one that's two then we have to write the total orbital angular momentum l which is the sum over the z components in the spherical representation which is zero for the s level or minus one zero or plus one for the p level so you have to sum all of these and if the result is zero then you write it as a capital s if it is one you write it as a capital p if it is two you write it as a capital d and so on and finally the j that is then where the spin orbit coupling appears there you tell how this l is oriented with respect to this s and this can take values from the absolute value of l minus s to l plus s. If we do that for these three levels with that single electron of sodium we find the three term symbols that you see there and in this way you see more clearly in the upper two levels the 2p three halves and the 2p one halves that there we have in both cases spin one and s one and orbital angular momentum one the p level but the j how the s is oriented with respect to the l that j is different one half or three halves and there are only two possibilities and if we now go to our blinking level which was the if I go back the 2p one half j equals one half so now we can see what is the correct situation if j equals one half if j has the minimal value then the l and the s must have opposite orientations so in this cartoon it would be picture C where the orbital angular momentum which we find with the right hand rule is pointing upwards has the opposite orientation than the spin angular momentum which is downwards so just to refresh some reasonings of working with multiplet symbols because we will need this when we will talk about the magnetic hyperfine interaction in two weeks from now there is a formalism there that is very much similar to the way how spin orbit coupling is described with these multiplet symbols. Some other nice pictures you have these two lines from sodium you can do classical physics experiments like the electric pickle there is a lot of vinegar in there and salt so you have sodium atoms around and if you put the proper voltage on this you can make your cucumber or pickle glow orange and that orange that is the same light that we have in the old highway lights nowadays many of the highway lights are being replaced by more daylight type LEDs on this picture you see both so soon this familiar physics example of orange sodium light will not be known to new generations of students anymore which is somewhat a pity but well so be it no daily life connection to fine structure anymore but we will always have gravitation so therefore let's look at a gravitational analog to the tools that we will need here as I explained in the videos why gravitation because this is classical physics and we can develop our mathematical machinery in a situation where we don't have to worry about quantum physics so it's good to focus on that mathematical machinery and only later we make this step towards quantum physics and the system we were discussing about that was two gravitationally interacting mass distributions a static situation we calculated the gravitational potential energy the full expression the general expression that's the red box you see at the left hand side and I asked you to calculate the same but now with the picture reversed where rather than starting from mass distribution one which generates and gravitational potential in which you put mass distribution two so no let's do it the other way around consider now mass distribution two that is generating a potential in which you put mass distribution one and not surprisingly if you calculate the potential energy in that way you find an expression that is exactly the same as the one we found before this is to illustrate that you can freely jump back and forth between a situation if I now talk about atoms again where you can jump back and forth from a situation where you describe everything from the point of view of the nucleus or from the point of view of the electron system and this is another cartoon way to make the same point if I have a classical hydrogen atom I can wonder what is the electrostatic energy of this that's a constant times the charge of the electron minus Q times the charge of the nucleus plus Q divided by the distance between both which is R in this circular orbit and the way how I have written it here this capital Q over R this looks like a potential generated by the nucleus and you put the energy in that potential and then you get sorry you put the electron in that potential and in that way you get the energy but you could also write it this way and this is just a trivial change but the interpretation becomes different minus small Q over R that's now the potential generated by the electron and you put the nucleus in that potential and obviously the formula is the same so you can jump back and forth between these two point of views. The second question in this context was we looked at this hypothetical mass distribution that you see here on the slide where we have variables R that measure positions inside the red mass distribution which will be the model for the nucleus and R prime these are positions in the blue mass distribution which is the model for the electron cloud and we discussed that we can make a multipole expansion out of that where we have these symbols are smaller than and are larger than which are either R or R prime or R1 or R2 depending on the notation and if we are in a situation where the R smaller than is always the nuclear coordinates and are larger than the electron coordinates if that is always true for any point in space then we can write this as pure multiples as a sum over dot products between pure multiple objects that where one of them only depends on nuclear coordinates and the other one only on electron coordinates and that will be a huge simplification. The confidence statement relating to this is I can write down an expression for the gravitational potential energy of two interacting mass distributions and explain why a multipole expansion of this expression is convenient. Why is this convenient you can calculate the nuclear and the electron part once and for all you do not need to know about one in order to calculate the other and it's written as a series expansion that is rapidly converging so you can take the first few simplest terms and work with these and that will be a very good approximation to the full picture. The other confidence question here was I can explain that issue of interpenetration of mass distributions which means when this R smaller than and R larger than do not always have a unique meaning and I asked you a specific question about that I asked you to draw examples of mass distributions where that condition was not fulfilled it's always good to know which situations are the difficult ones where the condition does not apply and what are the easy ones where the condition does apply where we can write the pure multipole expansion. Just a few answers that are not correct so on the left hand side I have two incorrect suggestions where you can see that indeed there can be situations where how is it called here where the R that even doesn't make much sense but what is wrong here in this example is that the origin of the two position vectors that one is not in the center of mass of the nuclear system because that was the convention we always made we want to have the origin in the center of mass of the object that will be the nucleus and that is clearly not the case here at the right hand side there is another issue you have two position vectors that start from two different positions and that is also not the case you take the axis system only once and all your positions are measured from that origin not from the center of mass of the nuclear system and the center of mass of the electron system separately. Then some examples that are correct well many examples that were correct and I can distribute them in three categories one set of examples is what you could call reverse the roles take a nuclear system that is here in white and an electron system that is barred and one is inside the other so the nuclear system is inside the electron system sorry the other way around the electron system is inside the nuclear system that's not what we usually do if we think about atoms but in principle in general because this was a question about mass distributions that is a perfectly legitimate solution. The second category is what I call the solution with bulges where you have a nuclear system that is usually inside the electron system just as would happen in an atom but there are some holes in the nuclear system and bulges in the electron system such that you can have situations where electron coordinates are smaller where position vectors for electron coordinates are smaller than position vectors for nuclear coordinates and on each of the examples you see here on the slide this is fulfilled and you have the possibility of what I call the category of long and flat if you have a very elongated nucleus then whether that nucleus is inside the electron system or next to the electron system like here at the upper right corner or here at the very left and also here this one these are nuclear and electron systems that are next to each other so that is one possibility or you can have the nucleus inside the electron system as here in the center but in all cases the nucleus is rather long and that can lead to situations where electrons are closer to the center of mass of the nucleus than some distant parts of the nucleus. So in all of these conditions in all of these examples the condition of r smaller than always being a nuclear coordinate is not fulfilled. From previous years I take some other examples where people explored three dimensional situations that do not fulfill that condition and you can see that in three dimensions you can have nice examples of this and I always show the one that I still consider the nicest example that has appeared over all those years and that is this one a very simple geometry where the nucleus is a bar and the electrons are a kind of ring and you can readily imagine in gravitation this could be two galaxies that are yeah that have these particular shapes and that are gravitationally interacting with each other so it is not a really far fetched situation so such a configuration there that multipole expansion will not be possible. So what is the summarizing point of this exercise? For nuclei and electrons we are used to making the multipole expansion much of atomic physics is just the first term in that multipole expansion but that multipole expansion is not general it applies only to these situations where r1 and r2 behave the way that we described. If you search for examples where you have something where that condition does not apply that is possible you have seen all these many possibilities and they do not need to be too far fetched and now in the next weeks we will realize that for real atoms that this condition does not strictly apply so a real atom cannot be expressed by a pure multipole expansion there will be effects due to violating that condition and that is part of our hyperfine effects not all of them there are hyperfine effects that are described by the pure multipole expansion but there will be also hyperfine effects that are described by these correction terms that appear due to violating this easy condition. And the last task for that part was a task where a few people were surprised about I asked you to calculate the electric monopole moment for these three charge distributions and some people wondered but isn't that something we have done already last week why do we have to repeat that well it has a reason and we will see the reason when once we have gone through the solutions so calculating the monopole moment here was very straight forward I show here two parts of your reports where people give the correct answer the monopole moment is just the total charge and in each of these three examples the total charge was called Q so therefore the monopole moment is Q in all three cases the same value same monopole moment. The second part of the question that was a bit more involved there I asked to calculate the mean square radius for each of these three charge distributions and I wrote I read in one report and somebody noted I don't understand the formulas for this situation too well well let us see at some of you who have written the explicit formula and I show here first this example the mean square radius for these two charges that are on the vertical axis the general expression for the mean square radius is what you see at the top of the slide this involves integrals over continuous charge distributions but we have discrete charge distributions here so we can reduce our integral to a sum and you see the explicit formulation on this slide and then you so you find the numerator of this ratio here as a squared times Q over 4 you have to divide that by the integral over the charge distribution which is the total charge so you have to divide this by Q and the mean square radius turns out to be a squared over 4 where a is the distance between these two charges. I enlarge here the important formula the general formula for the mean square radius and the version the discrete version if you have not a continuous charge distribution but a collection of point charges so this is a correct answer but I want to stress that this answer depends on the choice of axis system so we have calculated here the right hand side where the origin of the axis system was taken in the center between the two charges and their distances from the origin were therefore a over 2 if I go back to this expression you see here the minus and plus a over 2 this is where you see that this origin has been chosen and it's written here we choose the origin right in between the two particles if you would make a different choice if you would take the origin in one of the particles then the mean square radius will be different a squared over 2 so mean square radius is a property that depends on the choice of axis system and if we talk about the mean square radius of a nucleus and that is something we will use in the coming weeks then we implicitly assume that this is for an axis system where the origin has been taken in the center of mass of the nucleus. The other charge distribution was this triangle where the edges of the triangle were given the symbol a and therefore the distance from the center of mass of the triangle is a over square root 3 and if you then go through the calculation you find that with an origin of an axis system that is in the center of mass the mean square radius is a squared over 3 and the full calculation taken from one of your reports is here and for the two different mass distributions always with the origin choice in the center of mass of these two mass distributions. If you go to the third mass distribution there the integration is over a continuous mass distribution now and if you want to see how that is done you have the short version of the derivation at the top of the slide. The point I want to make here and that is what this exercise was meant for we had here three mass distributions or charge distributions with the same monopole mode always Q but the mean square radius that can be different we have a squared over 4 a squared over 3 and again a squared over 4 so nuclei with the same charge with the same monopole moment can have different mean square radii these are independent properties if one is the same if the monopole moments are the same that doesn't mean that the mean square radii need to be the same that is the reason of going through that exercise. And I also asked you what is can you give examples of principle axis systems and that was another topic we needed here in this situation axis systems in which objects like vectors or tensors can be described in the most simple way and you gave some examples about precessing tools where you can work most conveniently in an axis system that coincides with the symmetry axis of the precessing tool or the famous example here at the bottom of the slide the inertia tensor you can always find for any object an axis system in which the inertia tensor is diagonal so and that is the easiest axis system to describe that inertia tensor for that object. In classical mechanics you have other places where you meet principle axis systems somebody quoted the parallel axis theorem in solid state physics you have the interaction between polarization and magnetization where indeed a principle axis system can play a role. In gravitational problems if you have a planetary orbit that is an ellipse you can describe that ellipse in various axis systems but one of them is simpler than the others and that is the principle axis system for the ellipse or in three dimensions for the ellipse width you can have Hamiltonians that can be easier in one axis system than the other and that too has a relation to principle axis system so the concept of searching for an axis system where a given object becomes simpler that is as such a concept that you have met many times before. Meanwhile I keep keeping an eye on the chat every now and then I see no activity there but once again if I tell something on which you want to react put it in the chat and every now and then I will go back there and try to give answers or feedback on what you have put there. If nothing appears I just continue and that brings us to the double ring example which is still a gravitational example not for a general system but for a very specific system of two rings with mass and in the center between the two rings a dumbbell two masses connected by a massless rod and we look at the gravitational interaction between these two objects and we calculated the quadrupole correction to this so we made the multipole expansion this is a pure multipole here there is the condition that distances inside the dumbbell are always smaller than distances from the origin of the dumbbell to the rings this condition is certainly met here so we can make a pure multipole expansion and we look here at the quadrupole term that gives rise to the quadrupole energy that has for this system the expression that is written there a constant that depends on the geometrical details and that is given the name here alpha and then something that depends on the orientation of the dumbbell and we distinguished three situations where the geometry is such that alpha is positive or that alpha is zero or that alpha is negative and I asked you for each of these three situations try to deduce what is the lowest energy orientation of the dumbbell and don't calculate that you can of course calculate that but try to find visual arguments for that because that will help you to give meaning to this example and several people answered by arguments that I would say are derived from the formula so they are correct if alpha is positive and you fill out then some angles of theta then you see that the lowest energy orientation happens when theta equals zero so when the dumbbell is parallel with the z axis if alpha is zero then theta can be whatever it is there is no energy correction at the quadrupole level so all orientations are identical in energy up to the quadrupole term and if alpha is negative then it turns out that theta 90 degrees is the lowest energy solution that comes from the formula but how can you deduce this from the pictures if alpha equals zero then you have two rings where the diameter of the rings is relatively small compared to the distance between the rings and you can try to go to an extreme situation where you have very small rings that are very far away from each other so you have two point masses on the z axis that are interacting with that dumbbell and what do the masses want to do in such a situation they want to minimize the distance between each other so that can be done by putting the dumbbell vertical in the opposite extreme alpha negative we have very large rings that are very close to each other so in the limits this is just one large ring in these two large rings just are superimposed on each other and again masses want to minimize the distances due to the gravitational attraction and how can they do that by putting the dumbbell in the plane of the rings and then the situation with alpha equals zero there you have a special configuration in between where you make the cross over between these two pictures where the quadrupole interaction vanishes and I will not tell what is the geometric interpretation of that situation here later when we will be in the quadrupole topic in two or three weeks from now then you will understand what is the symmetry argument that makes that particular situation in the middle exactly zero at the quadrupole level. In this statement I can relate a free atom qualitatively to a double ring system with a dumbbell nucleus and I can explain up to the quadrupole turn how the orientation of the system affects the potential energy how the orientation of the dumbbell affects the gravitational potential energy of the system that is exactly what I just did so if you understand that reasoning then you can answer a six on this scale. The last topic of this week is the transition from the classical gravitational part to the quantum situation so what is a multipole expansion in quantum physics and how do we need to do something different than we did in the classical situation and here I ask you try to describe the complications you would run into if you would try to use perturbation theory to study a system with a nucleus of a general shape and I have drawn there a nucleus of a general shape so deliberately a weird non-symmetrical object so we want to describe a nucleus of a general shape in an atom but we don't make a multipole expansion first we immediately apply perturbation theory and I assumed here that perturbation theory is something you have met before for those who wanted to brush up their memory about perturbation theory there was a separate video in the refresher section to which I referred so I assume that the ones who needed to do that have done that so I can directly use perturbation theory and well what is one of your answers and how would perturbation theory look like in this situation well we first take the easy system the point nucleus and then we take everything that is not point like so graphically cartoon like what you see here on the slide the easy point nucleus and then the extra term that is complicated you can feel that if you want to do perturbation theory with that special extra term that looks so complicated that you will have difficult integrals to solve so using perturbation theory for a general nucleus right away without doing the multipole expansion that would be that would lead to difficult integrals that is the short version. Therefore to avoid these difficult integrals that's why we make the multipole expansion and that is what I try to illustrate with this question and with this example what is the actual reason to make the multipole expansion we want to avoid difficult integrals we represent our nucleus as a point that can have a radius so it can also be a sphere that can have an elongation so it can be an ellipsoid and higher order corrections that will be so small that we can neglect them up to now in most of your atomic physics and solid state physics you have just considered the point nucleus there was no sphere there was no ellipsoid by allowing the sphere by allowing the ellipsoid you get small energy corrections and that will be our hyperfine corrections so that is what it leads to. The second part so the first question was about perturbation theory the second one is about the multipole expansion so which approximations do you make if you do a multipole expansion when is the multipole expansion a valid expansion that can be valid in the sense you can easily truncate it only after a few terms it will be so small the extra terms will be so small that you can truncate it when does that condition appear so when is it valid to truncate the multipole expansion early in the series and I show here a few of your answers again if the further correction terms are much smaller than the leading order terms of the distribution when the higher order terms become negligible very fast yes I can agree with that but why when does that happen and that is given in the third answer when the distance between charges compared to the distance to the objects influenced by the field created by the charges is small that's a bit of a complicated way to tell when the typical position vectors of electrons are much larger in length than typical position vectors inside the nucleus if that condition is fulfilled then the multipole expansion will rapidly converge or without any mathematical statements if nuclear dimensions are much smaller than the dimensions of the electron clouds then the multipole expansion will rapidly converge and we know that that is true nuclear dimensions femtometers are orders of magnitude smaller than typical distances in the electron cloud which can be angstroms now going back to perturbation theory when can you stop at first order and I show here from the video on the on perturbation theory from that refreshers video I show here the general expression with zero to order first order and second order perturbation and you can see from this that when can you stop at first order so when is the second order term negligible that is when the denominator in the second order term is small sorry when the denominator is large because you divide by a large number and that denominator is large if the energy difference between two unperturbed levels is sufficiently large so if you are discussing fine structure energy levels and you want to find corrections to fine structure levels so these energies there will be energy differences between fine structures so millielectron volt energy differences if the effects we are describing and we know our hyperfine effects will be microelectron volts in size so that is 1000 times smaller so if the energy corrections you search for are smaller than the unperturbed energy distances then we can stop after the first order term that is what basically will happen with hyperfine interactions so first order perturbation theory will be okay. Can we then realize for a typical atom both stopping early with perturbation theory and stopping early in the multiple series can we satisfy both stopping criteria simultaneously that okay I just look at the time but we are still nicely within time can that be done yes because we concluded before that when the nuclear dimensions are much smaller than the electron dimensions then the multiple series rapidly converges and now we can use all the terms of the multiple series that we consider the ones we do not the ones we do not neglect for these are corrections for which we have to apply the perturbation theory when is perturbation theory small or when can you stop at first order perturbation theory that too will happen when the nuclear dimensions are smaller than the electron dimensions because it is in such a situation that you can have these relatively large fine structure energies and correspondingly small hyperfine energies. So an atom is an ideal system on which you can apply both the multiple expansion and truncate that quite early and first order perturbation theory in that expression for second order perturbation theory the denominator will be large and the numerator which are matrix elements of these higher order terms these will be very small. So to summarize that point what is the difference between the multiple expansion in a classical system the gravitational example and a quantum system the atom of the solid or the solid in a classical system the multiple expansion is used to make the integrals easy to make that classical system tractable computable if you want to do the same in a quantum system then making the multiple expansion alone is not enough you have to combine it with perturbation theory and the easiest situations arrive when you can have only the first correction terms in the multiple expansion and treat them at first order perturbation theory which is what we will do our hyperfine effects will come from this. And that is the end of this discussion as usual I wait for half a minute or so which is the delay of the video stream to see whether there is something appearing in the chat so if you have further questions that reminds me that there was one technical question to which I did not answer yet somebody mentioned yesterday that contributions to the forum for the multiple expansion apparently were not accepted that his or her comments did not appear in the forum I did not have the time to check that I just saw that there were many entries in the forum so at least for most people it works whether that particular person had well whether these contributions effectively appeared in the forum or not I did not check it but I will do that later today so I will then answer privately to confirm whether or not there is a problem there. I do not see any answer or any question in the chat okay so that means we can stop here and I'll see you next week when we will have our discussion on the electric monopole shifts bye bye